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Dynamical phenomena in the hierarchical fermionic model

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Abstract

We describe the global behaviour of stable invariant curves of renormalization group transformation in the coupling constants plane of the hierarchical fermionic model. These results are interpreted in terms of complex critical phenomena in our model. © 1999 Published by Elsevier Science B.V.

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In paper [1] we proposed a full fermionic analog of the hierarchical φ^4-model (see also work of Dorlas [2], where some degenerate hierarchical fermionic model is discussed). Using the local property of hierarchical renormalization group transformation, Bleher and Sinai [3], Collet and Eckmann [4] and others rigorously described the picture of critical phenomena in the hierarchical bosonic models. Renormalization group (RG) transformation in these models acts as a nonlinear integral operator in the infinite-dimensional space of density functions, giving probability distribution of single spin. Due to the Pauli principle for Grassmann-valued spins the RG-transformation in the hierarchical fermionic model is reduced to the rational map in the 2-dimensional coupling constants plane. This allows to give an exact description of many features of the generated dynamical system and even to visualize RG-flow. Our model can be considered as a lattice fermionic statistical system with rich critical properties. It is noteworthy that there is a continuous (p-adic) version of this model, which can be considered as a continuous quantum field system. As a dynamical system our model exhibits a wide variety of dynamical phenomena.

We'll begin with a brief consideration of some definition and results (see Ref. [1]).

Let $N = \{1,2,\ldots\}$, $V_{k,s} = \{j : j \in N, (k-1)n^s < j \leqslant kn^s\}$, $k \in N$, $s \in N$, and let $s(i,j) = \min\{s :$ there is k such that $i \in V_{k,s}, j \in V_{k,s}\}$. The hierarchical distance $d(i,j)$, $j \in N$ is defined as $d(i,j) = n^{s(i,j)}$, if $i \neq j$ and $d(i,i) = 0$. Let us consider the 4-component fermionic field $\psi^*(i) = (\bar\psi_1(i), \psi_1(i), \bar\psi_2(i), \psi_2(i))$, $i \in N$, where the components are generators of a Grassmann algebra.

The "Gaussian" fermionic field with zero mean and binary correlation function

$$\langle \psi_k(i)\,\bar\psi_l(j)\rangle = \delta_{k,l}b(i,j), \quad k,l = 1,2, \qquad b(i,j) = \frac{1-n^{1-\alpha}}{1-n^{\alpha-2}}\,d^{\alpha-2}(i,j), \quad i \neq j,$$

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